Today, I will take a topic I briefly introduced in the previous post on percentages. Let me start with the question I posted there.

What does a 20% sale with an additional 25% off on the $85 sweater that you have your eye on mean to you?

It means a big rebate. Let's see how much:

If you reduce 85 by 20%, it becomes 85 * 4/5 = $68. Now, you reduce it again by 25% and it becomes 68 *(3/4) = $51

Notice that a 20% discount and then a 25% discount is not equal to a 45% discount (85 * 55/100 = $46.75). It is less than 45% but to the imperceptive, oblivious customer, it registers as 45% (Now you know why the retailers use the strategy of marking down by 20% and then giving an 'additional' 25% later!). The difference arises because the 20% discount was given on $85 but the 25% discount was given on $68. 25% of 68 is definitely less than 25% of 85 and hence the overall percentage decrease is less than 45%.

This is called successive percentage change – a number is changed by some percentage and then *the new number* is changed by another percentage. Both the percentage changes are not applied to the same original number.

The most popular example of successive percentage change is population change. Let us look at an example to understand this.

Example 1:

A city's population was 10,000 at the end of 2008. In 2009, it increased by 10% and in 2010, it decreased by 18.18%. What was the city's population at the end of 2010?

Solution:

Population at the end of 2008 = 10,000

Population at the end of 2009 = 10,000 * (11/10) = 11,000

Population at the end of 2010 = 11,000 * (9/11) = 9000

Simply put, population at the end of 2010 = 10,000 * (11/10) * (9/11) = 9000

It is best to do the calculations in a single step because you do not need to calculate the intermediate population values. Besides, there is a good possibility that factors will get canceled out and hence, you will need to do fewer calculations.

Obviously, there is no limit to the number of successive percentage changes that can be made to a number. The approach remains unchanged in any case. Let me elaborate with another example:

Example 2:

Six months back, the cost of an air ticket from Detroit to San Francisco was $400. Four months back, the fares increased by 12.5%. Last month, the fares increased by 25% and yesterday, the airlines again increased the fares by 11.11%. What is the price of a Detroit to San Francisco ticket today?

Solution:

Price of a ticket today = 400 * (9/8) * (5/4) * (10/9) = $625

This is much faster than finding the ticket price at every price change which would need the following steps:

Price of a ticket four months back = 400 * (9/8) = $450

Price of a ticket last month back = 450 * (5/4) = $562.5

and finally, price of a ticket today = 562.5 * (10/9) = $625

So, in case you do not need the intermediate values, do not calculate them.

When there are only two successive percentage changes, we can derive a formula. In some cases, the formula makes the solution very simple.

When a number, N, changes by x% and then changes again by y%, we do the following to find the new number:

New number = N * (1 + x/100) * (1 + y/100)

Now, (1 + x/100) * (1 + y/100) = 1 + x/100 + y/100 + xy/10000

If we say that ***x + y + xy/100 = z***, then (1 + x/100) * (1 + y/100) = 1 + z/100

Here, ***z is the effective percentage change*** when a number is changed successively by two percentage changes. Let's take another example to see the formula in action:

Example 3:

A city's population was 10,000 at the end of 2008. In 2009, it increased by 20% and in 2010, it decreased by 10%. What was the city's population at the end of 2010?

x% = 20%

y% = – 10% (Notice the negative sign here because this is a decrease)

Effective percentage change = x + y + xy/100 = 20 + (– 10) + 20*(-10)/100 = 8%

Population at the end of 2010 = 10,000 * (108/100) = 10800

Note: When the percentage is a decrease, a negative sign is used as shown above.

This formula is used only when there are two successive percentage changes and the percentages are easy to work with e.g. 15% and 25%, -10% and – 30% etc.

With more than two successive percentage changes or trickier percentage values e.g. 11.11% and 18.18%, 9.09% and 6.25% etc, stick to the method shown above.

A major application of successive percentage changes in GMAT is the MarkUp-Discount-Profit questions. We will take that topic next week but I will leave you with a question to ponder upon:

If a retailer marks up his goods by 40% and then offers a discount of 10%, what is his profit%?